University of Maryland
Department of Aerospace Engineering
ENAE 647: Flexible Multi-body Dynamics

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1 General information

- Prerequisite: ENAE 646: Advanced Dynamics of Aero Systems
- Lectures: Mondays and Wednesdays 14:00 to 15:15, in Building JNP, Room 1202
- Office hours : Mondays and Wednesdays 10:00 to 12:00
- Office: 3182 Martin Hall (EGR). Phone: (301) 405-0328. Email: obauchau@umd.edu
- The mandatory homework format, see section 8
- The statement of academic honesty, see section 9
- Please consult the Canvas page on a regular basis
- To schedule an appointment, please contact me via email at obauchau@umd.edu
- Students with disabilities should inform the instructor as soon as possible so that appropriate arrangements can be made according to University policies
- It is the students responsibility to inform the instructor of any intended absences for religious observances in advance. Prior notification is especially important in connection with quizzes and final examinations.

2 Topics to be covered in this course

The following topics to be covered in this course:

1. **Review of Particle Dynamics.** Newton's Laws. Conservative forces. Contact forces. Newtonian mechanics for a system of particles. Coordinate systems. Differential geometry of curves and surfaces.
2. **Kinematics of rigid motion.** The representation of finite rotations: the geometric and algebraic descriptions. The vectorial parameterization of rotation. The representation of finite motion: the geometric and algebraic descriptions. The generalized vectorial parameterization of motion.
3. **Kinematics and kinetics of rigid bodies.** Relative velocity and acceleration. The angular momentum vector. Equations of motion for a rigid body. The principle of work and energy. Planar motion of rigid bodies.

4. **Analytical dynamics.** Lagrange multiplier method. Generalized coordinates. Constraints. Virtual displacements and rotations. The principle of virtual work for static problems. D'Alembert's principle. Hamilton's Principle. Lagrange's Equations of motion.

5. **Constraint equations in multibody dynamics.** Holonomic and non-holonomic constraints, Lagrange multipliers. Formulation and classification of joints in mechanics analysis; modeling of lower and higher pairs. Relative motions at joints. Flexibility in joints.

6. **Methods for enforcing kinematic constraints.** Maggi's method, the null space method, the index-1 method, the coordinates partitioning method, the penalty function method, the Baumgarte method, the Lagrange multiplier technique, the augmented Lagrangian method.

7. **Formulation of flexible bodies in multibody dynamics.** The floating frame of reference formulation. Small strain formulations. Geometrically exact cable, beams, and plate formulations. Inertial and material formulations.

8. **Finite element modeling.** Finite element tools for the geometrically exact formulation of flexible elements. Computational aspects in the representation and interpolation of rotation and motion.

9. **Numerical integration methods.** Numerical stability of the dynamic simulation of constrained multibody systems. The Hilber-Hughes-Taylor scheme; the generalized-α scheme. Computational schemes for nonlinear unconditional stability.

3 Grading Policy

The overall numerical grade for this course will be computed using the weighting factors shown in table 1.

10 Homework Assignments	40%
4 Quizzes	40%
1 Final exam	20%
TOTAL	**100%**

Table 1: Grade weighting factors

4 Homework

Homework will be assigned on a weekly basis. Homework is assigned on Wednesday's lectures and is due the next Wednesday, in class. Homework is a vital part of the learning process, **and 40% of your final grade.** To make sure no homework is *forgotten*, an aging factor will be built into the grading as shown in table 2. Do not forget to use the mandatory homework format, see section 8.

5 Quizzes and Final

Four one-hour quizzes will be held at the dates posted on the schedule below. Quizzes are closed book and closed notes, but open minds. Three crib sheets, hand written front and back, are allowed

Date homework is turned in	Aging factor
On the due date	actual grade
Up to the Monday after due date	actual grade - 1/10
Up to the Wednesday after due date	actual grade - 2/10
Never turned in	0/10

Table 2: Aging factor for homework

at each quiz. The format of the comprehensive final is identical to that of the quizzes, but five crib sheets are allowed. The comprehensive final exam will be held on Thursday, Dec. 17^{th} 13:30-15:30.

If you cannot take a quiz or the final examination on the announced date, please contact the instructor *ahead of time*, so that alternative arrangement can be made.

6 Reference books

The textbook by Bauchau [1] will be used for this course. The following reference textbooks are a good source of information for this class and are on reference in the library.

- For dynamics: Lanczos [2], Kane [3], Meirovitch [4], Baruh [5], Haug [6], Ginsberg [7], Bhat and Dukkipati [8]
- For multibody dynamics: Josephs and Huston [9], Nikravesh [10], Amirouche [11], Pfeiffer [12], Géradin and Cardona [13].

7 Course Schedule

Wk.	Dates	Reading Assignments	Notes
1	Aug. 31^{st} - Sep. 4^{th}	Chapters 1 & 3	
2	Sep. 7^{th} - Sep. 11^{th}	Chapter 3	Labor day: Sep. 7^{th}
3	Sep. 14^{th} - Sep. 18^{th}	Chapter 4	**Quiz # 1:** Sep. 16^{th}
4	Sep. 21^{st} - Sep. 25^{th}	Chapter 5	
5	Sep. 28^{th} - Oct. 2^{nd}	Chapter 5	
6	Oct. 5^{th} - Oct. 9^{th}	Chapter 6	**Quiz # 2:** Oct. 7^{th}
7	Oct. 12^{th} - Oct. 16^{th}	Chapter 6	
8	Oct. 19^{th} - Oct. 23^{rd}	Chapter 7	
9	Oct. 26^{th} - Oct. 30^{th}	Chapters 7 & 8	**Quiz # 3:** Oct. 28^{th}
10	Nov. 2^{nd} - Nov. 6^{th}	Chapter 8	
11	Nov. 9^{th} - Nov. 13^{th}	Chapter 9	
12	Nov. 16^{th} - Nov. 20^{th}	Chapter 10	**Quiz # 4:** Nov. 18^{th}
13	Nov. 23^{rd} - Nov. 27^{th}	Chapter 11	Thanksgiving: Nov. $26\text{-}29^{th}$
14	Nov. 30^{th} - Dec. 4^{th}	Chapter 12	
15	Dec. 7^{th} - Dec. 11^{th}	Chapter 17	Last day of classes: Dec. 11^{th}
16	Dec. 14^{th} - Dec. 18^{th}		Final: Dec. 17^{th} 13:30-15:30

8 Mandatory Homework Format

The homework you turn in are a good measure of the quality of your work and the effort you put into a course. Make every effort to present your work in the best possible manner.

1. **PRINT** your name clearly on the **TOP RIGHT-HAND** corner of each and every page. Work only on one side of the sheets. STAPLE all sheets together.
2. Be careful about neatness and being organized. Please use *pencil and clean erasure*, **ink is not acceptable**.
3. **Neatness counts**, it is a sound approach to engineering.
4. Clearly indicate your final results by putting a **box around your answers**.
5. Do not forget to keep track of **UNITS** during your calculations.
   (a) It is a convenient way to *uncover math errors*.
   (b) Please indicate units for your final answers: **a final answer without units is not an answer.**
6. **Describe your solution process first:**
   (a) Provide a *sketch of the problem* with all relevant information.
   (b) If applicable, **draw a free body diagram.**
   (c) Derive the *equations of the problem.*
   (d) Discuss your *approach to solving* them (analytical, numerical, etc).
   (e) Provide expressions for all the results you are plotting.
   (f) Comment on the **physical nature and significance of your answers.**
   (g) Please do not give an answer that makes no sense! It clearly indicates you do not understand the material.
7. A plot conveys abstract information in a graphical manner. A **plot must be drawn with the help of a software package** (Matlab, Maple, etc.). Freehand drawing is a sketch, not a plot. Consider the plot shown in fig. 1.
   (a) Both x and y axes must be **properly labeled**; provide **UNITS**. If it is a non-dimensional quantity your are plotting, say so.
   (b) If more than one curve appears on the plot, make sure to **clearly differentiate them**. Use *different line styles or symbols.*
   (c) Provide a **caption** that explains what quantities are plotted along the x and y axes. If more than one curve appears on the plot, provide a **legend** to explain the *meaning of each curve*.
8. If you are using a software package (Matlab, Maple, etc.) as a part of your solution process, **include the input/output files to these packages as part of your submission**.

9 Statement of Academic Integrity

The University of Maryland, College Park has a nationally recognized Code of Academic Integrity administered by the Student Honor Council. This Code sets standards for academic integrity at Maryland for all undergraduate and graduate students. As a student you are responsible for upholding these standards for this course. It is very important for you to be aware of the consequences of cheating, fabrication, facilitation, and plagiarism.

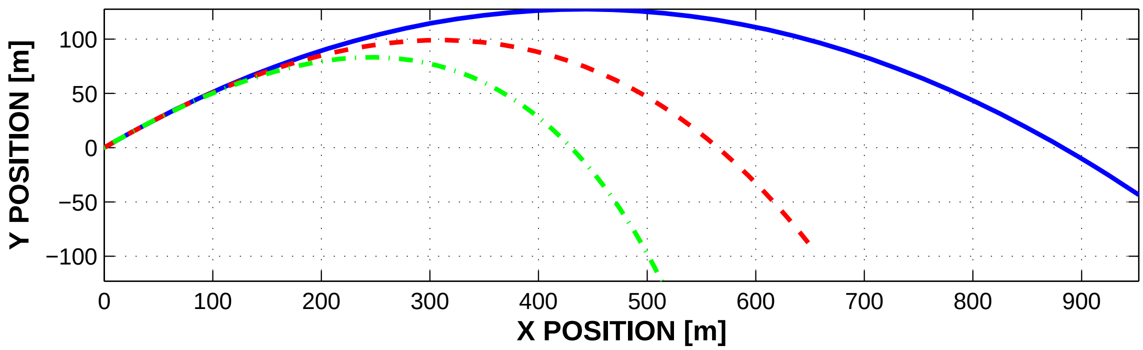

Figure 1: Trajectory of the particle: ignoring air friction (solid line); with friction coefficient k = 0.001 kg/m (dashed line); and k = 0.002 kg/m (dashed-dotted line).

You may, and are encouraged to discuss how to work out the problem sets with your classmates. Classmates are an excellent source of information. *There is an obvious difference between a constructive discussion about a homework with a friend and copying your friend's homework.*

References

[1] O.A. Bauchau. *Flexible Multibody Dynamics*. Springer, Dordrecht, Heidelberg, London, New-York, 2011.

[2] C. Lanczos. *The Variational Principles of Mechanics*. Dover Publications, Inc., New York, 1970.

[3] T.R. Kane. *Dynamics*. Holt, Rinehart and Winston, Inc, New York, 1968.

[4] L. Meirovitch. *Methods of Analytical Dynamics*. McGraw-Hill Book Company, New York, 1970.

[5] H. Baruh. *Analytical Dynamics*. McGraw-Hill Book Company, New-York, 1999.

[6] E.J. Haug. *Intermediate Dynamics*. Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1992.

[7] J.H. Ginsberg. *Advanced Engineering Dynamics*. Cambridge University Press, Cambridge, second edition, 1998.

[8] R.B. Bhat and R.V. Dukkipati. *Advanced Dynamics*. Narosa Publishing House, New Delhi, 2001.

[9] H. Josephs and R.L. Huston. *Dynamics of Mechanical Systems*. CRC Press, Boca Raton, 2002.

[10] P.E. Nikravesh. *Computer-Aided Analysis of Mechanical Systems*. Prentice-Hall, Englewood Cliffs, New Jersey, 1988.

[11] F.M.L. Amirouche. *Computational Methods in Multibody Dynamics*. Prentice-Hall, Englewood Cliffs, New Jersey, 1992.

[12] F. Pfeiffer and C. Glocker. *Multi-Body Dynamics with Unilateral Contacts*. John Wiley & Sons, Inc, New York, 1996.

[13] M. Géradin and A. Cardona. *Flexible Multibody System: A Finite Element Approach*. John Wiley & Sons, New York, 2001.